The Arithmetic Teacher

OCTOBER · 1955

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GLADYS and JOEL RHEINS

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A Journal of the
THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

THE ARITHMETIC TEACHER

a journal of
The National Council of Teachers of Mathematics

Editor: BEN A. SUELTZ, State University Teachers College, Cortland, N. Y.

Associate Editors: ESTHER J. SWENSON, University of Alabama, University, Ala. JOHN R. CLARK, New Hope, Pa.

All editorial correspondence, including books for review, should be addressed to the Editor. Advertising correspondence, subscriptions to THE ARITHMETIC TEACHER, and notice of change of address should be sent to:

The National Council of Teachers of Mathematics
1201 Sixteenth St., N.W., Washington 6, D.C.

Entered as second-class matter at the post office at Washington, D. C., June 21, 1954, under act of March 3, 1879. Additional entry granted at Menasha, Wis.

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THE ARITHMETIC TEACHER is published six times per year, in October, November, December, February, March, and April. The individual subscription price of $3.00 ($1.50 to students) includes membership in the Council. Institutional Subscription: $5.00 per year. Single copies: 50¢ each. Remittance should be made payable to the National Council of Teachers of Mathematics, 1201 Sixteenth St., N.W., Washington 6, D. C. Add 25¢ per year for mailing to Canada, 50¢ per year for mailing to other foreign countries.

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THE ARITHMETIC TEACHER

Volume II Number 3

October 1955

A Comparison of Two Methods of Compound Subtraction

THE DECOMPOSITION METHOD AND THE EQUAL ADDITIONS METHOD

GLADYS B. RHEINS,* *Great Neck, New York*
JOEL J. RHEINS,* *New York, New York*

Introduction

Present Status of the Subject

THE LAST DECADE has seen the movement for teaching arithmetic meaningfully become dominant in American schools. In consonance with it, a rationale has developed for teaching the processes of arithmetic in accordance with its basic concept.

With respect to teaching compound subtraction there is, however, a controversy as to which method, that of decomposition or that of equal additions, produces the better result. In their recent book K. Lovell† and C. H. J. Smith in England advocate the equal additions method.

> Considerable research has been made to decide which of these methods—subtraction by decomposition or subtraction by equal additions—is the better one to use. Ballard (England) maintains that with boys and girls the method of equal additions is quicker and fewer errors are made, although the advantage becomes less as the child becomes older (by about thirteen years of age). McClelland (Scotland) claims that the method of equal additions is superior in speed and accuracy although the method of decomposition is superior in speed after very long practice; while Johnson (U.S.A.) states that the decomposition method produces eighteen per cent more errors and requires fifteen per cent more time than the method of equal additions. The method recommended to be taught, then, is that of equal additions, for with younger children the mass of opinion is that although it is less easy to demonstrate in concrete form, it (a) produces fewer errors; (b) takes less time; (c) is less awkward to work when the minuend has one or more zeros in it.

On the other hand, R. L. Morton[9] and L. J. Brueckner[2] and F. E. Grossnickle advocate the method of decomposition in their recent books. The experimental evidence on which they base their recommendations is not sufficiently conclusive as to lead to an incontestable preference. In consequence they base their preference for the decomposition method on this reason—the decomposition method can be developed in accordance with the rationale of meaningful arithmetic much more easily than the method of equal additions.

H. E. Moser,[11] however, proposes additional research to determine which method of subtraction is the better.

Traditionally, subtraction has been taught in or by the third grade. By the

* Mrs. Rheins is a teacher of mathematics in the junior high school grades, and Mr. Rheins is an Assistant Principal at the James Fenimore Cooper School in New York City.

† Numbers refer to numerical references in the Bibliography.

eighth grade the process of subtraction has become largely automatic—a tool to be used in other processes, such as long division and in the solution of problems. It would be of value to know which technique of subtraction provides the more serviceable tool.

Aim of the Investigation

The purpose of this investigation is to determine whether the method of decomposition or the method of equal additions is superior in speed and accuracy in compound subtraction of whole numbers in abstract and in concrete examples, not immediately after the process has been taught, but after a time lapse of five years. The subjects are still in school and thus have used the process continuously since learning it for the solution of problems and as a tool in other processes notably in division. The factor of forgetting or loss in competence through disuse is in this case avoided.

Definition of Terms

By the method of DECOMPOSITION is meant the process by which an example of the type 43−27 is worked by adding 10 units to the 3 of 43; then subtracting 7 from 13 leaving 6; these ten units were taken from the 4 tens leaving 3 tens. Then 2 is subtracted from 3 leaving 1. The usual algorism for this process including crutches to show the thinking is as follows:

$$\begin{array}{rr} 3 & \\ \not{4} & {}^{1}3 \\ -2 & 7 \\ \hline 1 & 6 \end{array}$$

This method is called the method of decomposition because the number 43, composed of 4 tens and 3 units, was decomposed and then reconstituted into 3 tens and 13 units.

The method of EQUAL ADDITIONS is based on the principle that if equal amounts are added to two numbers, the difference between them remains the same. Hence 10 units added to 3 give 13. 13−7 leaves 6. 10 units are the equivalent of 1 ten. 2 tens plus 1 ten equal 3 tens. 4 tens−3 tens equal 1 ten. The usual algorism for this process including crutches to show the thought processes is as follows:

$$\begin{array}{rr} 4 & {}^{1}3 \\ -{}^{3}\not{2} & 7 \\ \hline 1 & 6 \end{array}$$

In speed tests it is usual to count the number of correct responses within a given period of time. In this investigation the number of examples done correctly within the time limit allowed was considered to be a measure of the speed of computation. The number of subtraction elements within the time limit was not counted as needlessly complicating the investigation. This was compensated for by arranging the test items cyclically. (See Test B.)

In accuracy it is usual to count the number of test responses that are correct out of the total number of responses. In the present investigation sufficient time was allotted to the test for accuracy to enable all subjects to complete and check all the test items. The items were arranged hierarchically so that the element of practice might operate from the simpler to the more difficult examples.

The speed test consisted of abstract examples in which whole numbers were subtracted from whole numbers.

The accuracy test consisted of two parts, Test A, in which the numbers to be subtracted were explicitly set forth as in abstract situation; and Test C, in which concrete situations were presented in the form of problems to be solved. Only whole numbers were used throughout the investigation.

Experimental Procedure

Selection of Pupils

One of the authors, a teacher of mathematics in Great Neck Junior High School,

has a program consisting of five eighth-year mathematics classes. It was proposed to draw all subjects from these classes. Each pupil was interviewed separately, using the example previously referred to on Page 64, to determine which method of subtraction the pupil used. The pupil's name was entered on an index card, keyed with the letter D or E (for decomposition or equal additions). The age and I.Q. of each pupil of the E group were also entered on the index cards. Then subjects from the D group were selected to match, in a one to one correspondence, the subjects in the E group. In all only 16 subjects in the E group were found. Thus there were available for the study 16 pairs of pupils matched in the following elements: sex, age, I.Q., school grade, and socio-economic index. The latter was an automatic result since the community of Great Neck is a suburb of New York City inhabited by the upper middle class. These pupils were also all white.

It was considered that 16 paired elements might not be sufficient to give reliable results. Therefore another 19 pairs of pupils were selected from another school. The pairing was done for the same elements in the same manner. In this case, however, the socio-economic index of all the pupils is very low middle class shading into the lower class. The pupils in this group are drawn from the eighth grade of a lower central Harlem junior high school (The James Fenimore Cooper J. H. S., 120 Manhattan), in New York City; and are all Negro. Thus it is possible by using the method of equal pairs to control the situation so that only a single variable is under investigation.

Construction of the Tests

There is a hierarchy of difficulty in subtraction examples ranging from the basic combinations (which show relative difficulties among themselves), decade combinations, compound subtraction, zero difficulties, and the number of elements per example. An example, such as 620,071 −52,334 shows difficulties involving decade subtraction, successive compound subtraction, consecutive zeros, and a deficiency in number of integers in the subtrahend.

Permission was received from the Silver Burdett Co. to use the subtraction examples on Page 359 of "Making Sure of Arithmetic" by Morton *et al.* as the basis for construction of the tests. In some cases liberties were taken with the numbers; in others, the order of the examples was changed. The chief value for this work was the progressive order of difficulty of the examples as indicated on the original test. The problem test was constructed with situations within the understanding of eighth grade pupils. In two out of the twelve problems, extraneous numbers (not involved in the computation) were used in order to prevent mechanical computation and to insure the problem's being read.

Test items were arranged on the test sheets for ease of scoring, leaving sufficient space in all cases for computation. Scoring of Test A and B was done with the aid of oaktag scoring strips. Test C was scored with the aid of an oaktag window card.

Time limits for the test were found by giving the test to several pupils not in the paired groups, in a study hall of which the author was in charge. The time limits decided upon were eight minutes for Test A with twenty-two items; four minutes for Test B with thirty items; and twelve minutes for Test C with twelve items (problems)—a total of twenty-four minutes. This was just long enough to enable the test to be administered with ease in a forty minute period.

Administration of the Tests

The test was given to all members of the author's classes in a regular class period. The pupils were not informed in advance that a test was to be given. The

pairs of subjects did not know that they were the special objects of investigation. Papers of the subjects were then withdrawn from the totality, scored and paired. The Harlem group was tested in the same way.

Summary of Data

Comparability of Groups

By reference to Table 1, it will be noted that most of the test subjects in both groups were thirteen year-olds—a normal situation for eighth graders. With a mean I.Q. of 109 and a standard deviation of 16, approximately 70% of the Great Neck group possessed I.Q.'s between 93 and 125; whereas 70% of the Harlem group were between 79 and 99 since the group had a mean I.Q. of 89 with a standard deviation of 10.

TABLE 1

A COMPARISON OF THE AGES AND I.Q.'S OF THE D AND E SECTIONS OF GREAT NECK AND HARLEM GROUPS

Group	Age		I.Q.	
	Mean	S.D.*	Mean	S.D.
Great Neck D	13^6	0^6	109	16
Great Neck E	13^5	0^7	109	16
Harlem D	13^8	0^7	89	10
Harlem E	13^6	0^7	89	10

* S.D. means Standard Deviation of the Mean, sometimes expressed as σ_M.

In testing† for the significance of differences between the means, it was found that the groups of subjects for both methods of subtraction were strictly comparable in that there was no significant difference between them as far as could be ascertained using the 5% level of confidence. These findings are presented in Table 2.

† All calculations in this paper are in accordance with the theories presented in Lewis, E. E. —"Methods of Statistical Analysis in Economics and Business"—See Chapters 5 and 6, pp. 187–249.

TABLE 2

CRITICAL RATIOS FOR AGE AND I.Q. FOR THE GREAT NECK AND HARLEM GROUPS CALCULATED FROM THE DIFFERENCE BETWEEN THE MEANS OF THE D AND E SECTIONS OF EACH GROUP

Group	Critical Ratio*	
	Age	I.Q.
Great Neck D & E	.50	.09
Harlem D & E	.87	.16

* For the Great Neck group a critical ratio greater than 2.13 and for the Harlem group a critical ratio greater than 2.10 would be significant.

Test Results

An inspection of the test results summarized in Table 3 seems to indicate that the subjects who used the decomposition method were more accurate than those who used the equal additions method. The results for speed and for problem solving seem inconclusive since the D section was superior in Harlem but inferior in Great Neck.

TABLE 3

MEAN SCORES ON THE TESTS OF SPEED, ACCURACY, AND PROBLEM SOLVING OF THE D AND E SECTIONS OF THE GREAT NECK AND HARLEM GROUPS

Group	Test A *Accuracy* Mean No. of Exs. Wrong	Test B *Speed* Mean No. of Exs. Correct	Test C *Problems* Mean No. of Exs. Correct
Great Neck D	1.1	17	10*
Great Neck E	1.5	18	10
Harlem D	1.7	17	8
Harlem E	3.9	15	7

* In rounding off, D section was rounded up to 10 and E section was rounded down to 10.

Statistical Analysis

An examination of the significance of the difference between the means of the results of the three tests for the D and E sections of the Great Neck and Harlem groups was carried out by two different

methods. These were: (1) finding the standard error of the difference of the two means; and (2) the paired method of testing a difference. The results of these calculations are shown in the following tabulations:

TABLE 4

THE SIGNIFICANCE OF THE DIFFERENCE BETWEEN THE RESULTS OF THE D AND E SECTIONS IN THE THREE TESTS USING THE PAIRED METHOD OF TESTING A DIFFERENCE

Group	Critical Ratios*		
	Test A	Test B	Test C
Great Neck D & E	.65	.83	.67
Harlem D & E	2.50†	1.21	1.47

* A critical ratio that exceeds 2.13 for Great Neck and 2.10 for Harlem indicates a significant difference in test results.

† Indicates superiority of "D" section.

TABLE 5

THE SIGNIFICANCE OF THE DIFFERENCE BETWEEN THE RESULTS OF THE D AND E SECTIONS IN THE THREE TESTS USING THE METHOD OF THE STANDARD ERROR OF THE DIFFERENCE OF TWO MEANS

Group	Critical Ratios*		
	Test A	Test B	Test C
Great Neck D & E	.67	.56	.69
Harlem D & E	2.22†	1.02	1.02

* A critical ratio that exceeds 2.13 for Great Neck and 2.10 for Harlem indicates a significant difference in test results.

† Indicates superiority of "D" section.

Tables 4 and 5 show that there is no significant difference between the two methods of subtraction with respect to speed or problem solving. With respect to the test for accuracy, there was no significant difference between the two sections of the Great Neck group; but there was a significant difference for the Harlem group in favor of the decomposition method. Both methods of calculation are in agreement.

Out of curiosity, the rank order of correlation between I.Q. and test scores was calculated for the Great Neck and Harlem groups. The results for Harlem and for Great Neck, as shown in Table 6, were quite dissimilar. The Great Neck group showed considerable correlation, while the Harlem group showed little correlation. It should prove of interest to investigate the causes of such different results.

TABLE 6

RANK ORDER CORRELATION COEFFICIENT BETWEEN INTELLIGENCE AND TEST SCORES

Group	"D"	"E"
Great Neck	.69	.44
Harlem	.12	.22

Conclusion

In this study the sections in each test group were strictly comparable. Furthermore, for the more intelligent group, the decomposition method was not inferior; whereas for the less intelligent group the decomposition method proved to be superior. Therefore, the conclusion is that the decomposition method of subtraction should be taught from the beginning of instruction in subtraction to all pupils. Moreover, the decomposition method is part of the rationale of meaningful mathematics, an integral part of its program.

The results of this study seem to bear out the arguments of the proponents of developmental mathematics that teaching meaningfully produces superior results especially with duller pupils.

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Survey Test—Mathematics

Name..... Period Date.....
Last First

Test A

DIRECTIONS: You will be allowed 8 minutes to answer all the examples. (Do not skip any.) If you finish before the time is up, you may go over your work to check your answers.

(1)	7 −3	(2)	9 −4	(3)	13 − 7	(4)	15 − 8	(5)	85 −32
(6)	76 −42	(7)	72 −46	(8)	83 −37	(9)	925 −713	(10)	836 −612
(11)	753 −426	(12)	641 −317	(13)	926 −284	(14)	827 −194	(15)	341 −183
(16)	431 −282	(17)	8630 −6791	(18)	8720 −6892	(19)	7000 −1268	(20)	6000 −2179
(21)	830,062 − 42,435	(22)	620,071 − 52,334						

Test B

DIRECTIONS: Do as many of the following examples as you can. There are more examples than can be done in the time allowed (4 minutes). Do not skip any.

(1)	486 −432	(2)	856 −404	(3)	928 −178	(4)	719 −260	(5)	430 −282
(6)	820 −545	(7)	8522 −4039	(8)	5123 −1064	(9)	738,461 −678,298	(10)	400,328 −196,676
(11)	974 −831	(12)	964 −402	(13)	563 −248	(14)	386 −327	(15)	813 −620
(16)	680 −194	(17)	8752 −. 988	(18)	2657 − 743	(19)	999,041 − 42,352	(20)	644,752 − 33,523
(21)	838 −766	(22)	698 −559	(23)	427 −408	(24)	1500 − 698	(25)	7746 −7077
(26)	9222 −6133	(27)	8743 −7408	(28)	7094 −2397	(29)	987,080 −579,817	(30)	620,071 − 52,334

Test C

DIRECTIONS: Work the following problems. Do your work in the space provided at the right of each problem. Then write the answer in the Answer Box*. You will be allowed 12 minutes.

1. On Sunday 456 passengers took the bus from Port Washington to Manor Haven. On Monday only 238 passengers took the bus. How many more passengers were there on Sunday?

2. In the first 3 days 321 pupils joined the G.O. If there are 814 pupils in the school, how many more must join to have 100% membership?

3. The receipts of the annual school band concert were $924. The senior class sold $538 worth of tickets. How much did the combined other classes bring in?

4. There are 940 seats in an auditorium. 163 of these are in the balcony. How many other seats are there?

5. In a certain park the total area is 504 acres. There is a large lake surrounded by 329 acres of land. How large is the lake?

6. There are 2163 pupils in a junior high school; but the auditorium holds only 604 seats. 175 seats have already been sold to the senior class. How many more seats are available?

7. Out of 3,000 pupils who brought notices home, only 2,791 returned the receipt stubs. How many notices probably did not get home.

8. In a town of 8,000 people 2,576 are voters. How many non-voters are there?

9. If 158 notices were returned by the Post Office out of 4,329 mailed to its members by the Civic Improvement Association, how many members received the notice?

10. 6,710 ballots were cast in the election for Mayor. Of these all but 93 were valid ballots. How many valid ballots were there?

11. In a Republican stronghold on Long Island out of 60,030 votes cast the Republican candidate received 50,043. How many votes did his opponent receive?

12. Mr. Jones allowed $50,000 this year for the operating expenses of his business. If rent and salaries accounted for $30,250, how much was available for other uses, such as advertising?

EDITOR'S NOTE. The choice of method of subtraction has long been one of the big problems in education. Here is a research that throws more light on the situation. The results of the research do not favor one method strongly. Certain factors that may have a bearing on the results are the original learning experiences in subtraction and such other factors as may account for the very low correlations between intelligence and test scores. Perhaps our best conclusion to date is that pupils can learn to subtract satisfactorily by one of several methods. It is altogether probable that some pupils will do better both initially and later by one method and other pupils excel by a different method. It is very difficult to set up an experiment that will answer all our questions regarding subtraction. We ought to consider such factors as: (a) the ease of learning the process at both the initial and later learning stages, (b) the facility and depth in understanding of the mechanics of the process, (c) the ease of fitting the process with numbers to situations involving subtraction and vice versa, (d) the accuracy of results at all stages from early childhood to later years, and (e) the usefulness and applicability of the method used with whole numbers and later transferred to fractions, measures, and denominate numbers. Are there other recent researches that shed light on subtraction?

* Answer boxes which appeared on the original test have been omitted for ease in printing.

Arithmetic Experiences in Grade One

DOROTHY JACK,* *Seattle, Washington*
AMANDA HEBELER,* *Ellensburg, Washington*

ONE MORNING Marie announced to her classmates, "We have a new boy baby. Now we have three boys and three girls."

Marie was praised for her number story then asked if she knew what number fact her story told.

Someone in the class was able to give the correct number fact. The result was a chart story which read as follows:

MARIE'S NUMBER STORY

We have a new boy baby.
Now we have three boys.
And we have three girls.
Three and three are six.

This story was followed by other number stories as the summer progressed. The children were encouraged to tell their number stories but the distinction between a number fact and a number story was the important element in judging which were to be recorded.

Some of the children were able to give several number combinations. However to put these same combinations in a setting to make a number story required organization of ideas which was much more difficult for them.

The content of the number stories was varied and typical of the interests of first graders.

Here are a few examples:

STERLING'S NUMBER STORY

Before I was born there were
three people in my family.
When I came along there were four.
Three and one are four.

LINDA'S NUMBER STORY

Grandmother gave me three bracelets.
Two of them are alike.
One is different.
Two and one are three.

TOMMY'S NUMBER STORY

The dentist pulled two of my teeth.
Two teeth were coming in behind them.
Two and two are four.

This phase of our number program resulted in training in language usage, association of arithmetic and beginning reading activities and might be considered an important first step toward comprehension of story problems.

* Miss Jack serves as consultant in the Seattle public schools and Miss Hebeler is Director of the College Elementary School at Ellensburg. This article is a report of work at Ellensburg where Miss Jack served as supervising room teacher during the summer.

The second phase of our number program was concerned with counting, After rote counting which all of the children could do easily the plan was to work out a device which would point up for the children the place of 10 in our number system.

A large pocket chart made of "butcher paper" with ten rows of pockets into which tongue depressors fitted was the first counting device used. The tongue depressors were painted blue for contrast with the white background of the chart.

In using this device the teacher would ask for a volunteer to display a card with a number. (See photograph.) For example when number 19 was shown the relationship of ten and nine was pointed out. Tommy placed a row of 10 counters in the first row and in the second row 9 counters.

Sometimes a child would give a number orally, a second child would be asked to write the number on the blackboard. Then a third child would show the number with the correct number of counters.

The children were guided to put ten in each row and the number which told what portion of ten remained in the following row.

This gave a visual representation of the place value of numbers in the decimal system.

To vary the counting experience round counting disks were used. Each child was given a small paper plate containing 40 of these disks. One child was chosen to be the leader. He selected a card with a number. This he held in plain view of the children participating.

This number was then counted out by placing the disks in rows of 10 from left to right. For example if 34 were shown, three rows of ten each and a fourth row of four counting disks were placed on the table in front of each child.

To check the learning, individual children were called upon. Each child in turn would point to the first row of ten disks placing his finger upon the tenth and say "10," pointing to the second row again placing his finger on the last disk say, "20" then quickly add any disks remaining in the following row which did not make a total of ten.

The child reporting his result would say, for example, "10–20–30 and four making a total of 34."

This and other types of practice placing boys and girls, disks, or other objects in groups of ten helped the children in developing the concept that numbers above 10 are composed of one or more groups of ten and units of one.

This was made clear by frequent use of the pocket chart which seemed to be the most effective device for showing one-to-one and one-to-ten relationships.

In summarizing, there are two obvious conclusions:

First, early counting experiences should involve association with actual objects and will be most profitable when these experiences are carefully directed from the beginning.

Second, the merit of any number game should be judged by the actual arithmetic learnings gained through the active participation of the class in real experiences with concrete objects.

Another consideration would be the ease of checking by the teacher for accurate responses.

EDITOR'S NOTE. This example of using children's normal interests as a medium for counting and learning simple number combinations shows how a thoughtful teacher plans and leads the pupils toward understanding numbers and the use of number symbols in our decimal number system. The story can easily be continued and extended. It is a fine art to know when and how to lead and direct as a teacher toward desired goals.

Jersey City Arithmetic Conference

On October 22, the Teachers College at Jersey City, New Jersey will hold an arithmetic conference which will feature demonstrations in third grade and fifth grade arithmetic. Dr. John R. Clark and Dr. Leo J. Brueckner will discuss and evaluate the lessons.

The Function of Charts in the Arithmetic Program

CATHERINE M. WILLIAMS
Ohio State University, Columbus

MANY teachers do an effective job of using concrete manipulative materials to help children with initial stages of the formation of arithmetic concepts. All too frequently, however, the teachers shift so rapidly from these concrete, visual experiences to abstract verbal symbols that children are unable to breach the gap. Children need intermediary helps paced to their emerging insights. Charts are one type of visual aid which can be developed to help children along the road from need for concrete materials to ability to operate with abstractions.

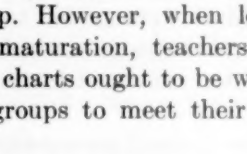
Usually children should participate in the development of the chart. This participation fosters the habit of organizing one's own ideas and of resourcefully working out one's own self-helps. Then, too, participation in developing a chart serves as insurance against introduction of material in chart form before the children are ready for the organized presentation. Because the pupils understand a chart which they have helped to work out, it becomes the meaningful reference which they need temporarily.

One of the common mistakes in the teaching of arithmetic is that charts (this includes all types of tables) which appear in arithmetic texts are introduced before experience has made them meaningful. Not even the most simple of organizational charts should be introduced until after concepts have been developed through rich first-hand experiences. This seems obvious when we remember that the sense, the meaning of any logically organized form is a development process which evolves as experience provides new insights. Since the meaning of any logical organization is an outgrowth of one's own experiences, children should not be denied opportunities to build up their own organization. After children have worked out their organization, they have basic understanding and are ready critically to study the mathematician's mature organization. Only then will they be able to appreciate his refined terms and short-cuts and/or his inclusion of information which they may have overlooked or lacked.

In this article it will be possible to explore only a limited number of situations wherein the teacher-pupil developed chart can be an important aid to the learning of arithmetic. Often chart-making situations can profitably include all members of a class group. However, when learning is paced to maturation, teachers will find that some charts ought to be worked out with sub-groups to meet their readiness needs.

Developing Charts

Teachers have discovered that beginners master forming letter symbols more readily when the manuscript alphabet is exhibited for their reference. Comparable guidance for the formation of figure symbols is seldom provided. However, as children learn the number symbols, teacher and pupils together might well work out a helping chart. For such a chart black crayon might be used for making the symbols. Red crayon might well be used for arrows which would indicate the path the pencil should take. Perhaps a dotted line could be used to indicate the part to be made last, for example.



Sometimes charts should not be completed all at once but should evolve cumulatively as new experiences contribute further pertinent information. A simple chart of this type might bear the title, *Signs We Should Know*. Signs of operation, the equal signs and the dollar sign are among those which would have a place on this chart.

When children reach that stage of maturity where familiarity with the terms for designating the numbers involved in each of the four fundamental processes is desirable, a chart might be developed. This chart would include a simple example of each process, accompanied by the proper term for designating each number. This chart would become a useful reference until such time as children are thoroughly familiar with all of the terms.

The three charts already mentioned typify the sort of information which is usually introduced with care, but which becomes unnecessarily complicated for children because they are not provided with helps to tide them over until the information is firmly established. The child for whom the coordination required to form the figures is a struggle should not be denied adequate reminder concerning the path the pencil should take. Children who are uncertain of which operation is indicated or those for whom memory fails and terms are a matter of guesswork cannot develop the sort of confidence which engenders further success. We teachers are short-sighted, indeed, if we fail to develop with children reference materials so that they can confidently and resourcefully undertake their tasks. Given such helps along with encouragement to become independent of these aids, children will delight in reporting, "I don't have to look anymore," or "I'm going to try this time and not look even once."

Occasionally charts should be developed simultaneously with the initial learning experience. A chart, *Rectangles*, might be developed when children are learning to recognize and identify the rectangle. In this case actual objects in the classroom, the door, the windows, and the like are being studied and, if recognized as belonging, are listed. The children might next bring in cut-outs from periodicals and mount them on another chart, *More Rectangles*, thereby extending the number of objects recognizable as rectangular. After children have had similar experience with square, circle and other common geometric figures, teacher and pupils together might well summarize the information in a chart, *Shapes Have Names*. This chart would include the picture symbols accompanied by appropriate word symbols.

Another type of chart which should closely follow experience is illustrated by a picture chart of liquid measure. After young children have used water to fill and empty into one another of the standard measures, along with ordinary containers such as milk bottles, a simple picture chart *Liquid Measure* would serve to summarize the information so recently discovered and verified through the experience. The chart would serve also as a reference until such time as a reminder becomes no longer essential. When children are more mature, the pictures would be unnecessary. However, participation in working out the organization of tables of measure, following initial experience, can enrich the learning at all levels. A chart regarding the measurement of time for example should not be developed all at once. It should be an outgrowth of experiences which, perhaps, had their origin in the discovery that seven days equal a week. When week, month and year have meaning, their relationship might be worked out into a simple chart. Information gathered over a few school years is necessary before a complete table of time measures can be meaningfully developed. The advantage of pupil participation in organizing their own table-type chart obviously is not that they will develop a unique chart. The advantages in their participa-

tion are that the learners think through what belongs and work out where it belongs in the scheme. When this type of chart seems complete to the learners, they should be encouraged to compare their results with standard tables found in textbooks designed for their level of maturity. Children who have this sort of experience have been spared rote memorization of another's organization of information, *they have been guided to organizing their own information into a table.* This latter procedure affords a very different quality of experience for the learner.

Let us turn attention to developing insight and facility with various ways of grouping whole numbers. If purposeful learning experiences do not occur with frequency and variety, the teacher should supplement them with contrived experiences. Experiences should be planned to utilize a variety of concrete materials for children to think with, to experiment with, to use for verification which leads to formulation of generalizations about groups of numbers. When children demonstrate understanding as they work with concrete materials, they are ready to organize information in simple chart form. An example is a chart, *About 8,* made to show the addition facts about eight. For an easy-to-make chart of this type dark background satisfactorily sets off white, half-inch, self-sticking discs (Dennison's Pres-a-ply Labels). Not only does this chart summarize the various combinations which total eight but also it is useful in establishing the relationship between addition and subtraction.

Charts may well be developed to build understanding of a process, for example, division. A chart of this sort might grow out of a situation where children need to find how many five-cent items can be purchased with thirty-five cents. Children should be encouraged each to figure out his own way of arriving at the solution. One child may perform successive subtractions. Another may fence off dot pictures in this fashion .. / .... Another may use multiplication as an aid. When solutions have been arrived at, the teacher might ask each child to demonstrate the means by which he arrived at seven. The various ways should be studied and, perhaps, become a part of a chart *We Learn To Divide.* The various methods of arriving at solutions can help children to important discoveries. Experimentation will show that the new method, division, is shortest and fastest. At this point the teacher has an obligation to help children to realization of the fact that people lived for many years before anyone discovered this faster, shorter method called division. Children will appreciate mathematics if constantly they are helped to see that number is an ingenious system worked out by man as his insights developed. After several experiences with the new process, division, children should be ready to work out what additional information they think belongs on the chart, *We Learn To Divide.* Producing such a chart challenges children to self-reliant thinking and, because it builds both confidence and insight, serves as impetus for using the short-cut, division.

A group of charts which will aid in developing insight into the process of division consists of several helping charts. *Helps for Dividing by 7* would be but one of the series. Others would be developed by substituting various ones of the digits for the seven. To make the seven chart, mark off the paper into seven vertical spaces and ten horizontal spaces. The top rows from left to right would read

1	2	3	4	5	6	7
8	9	10	11	12	13	14.

Use of the chart is facilitated if the figures which are multiples of seven are shown in color. Such a chart helps to build understanding especially when remainders are involved. Confronted by the example, $7\overline{)45}$, children will recognize 42 as the

product nearest 45. They will readily arrive at the quotient, six, with three remainder. Later when, children need to transfer remainders into fractional parts, this chart enables them *to see* that the three is three toward the next seven or three-sevenths.

When children are learning to understand large numbers both whole and those involving decimal fractions, they need a great deal of guided experience designed to build insight into value dependent upon position. When children experience a large number the teacher has both opportunity and obligation to help them to some realization of what such a number really signifies. A case in point occurred in a fourth grade engaged in a study of prehistoric animals. The earth was found to be about two billion years old. The teacher inquired if anyone could write two billion in numbers and a volunteer did so correctly. Then she suggested that the group might work together to find out how much two billion is. They worked out a chart *This Is Two Billion*, by starting with ten, $10 \times 10 = 100$ and so on. In this manner the teacher helped to extend the tens concept and to build understanding of position as well as to show children a good way of attempting to grasp the meaning of a large number.

Another type of chart, one showing the composition of a number, is also helpful in developing understanding. For example, the number 630,734 is partly composed of:

6 hundred thousands	600,000
3 ten thousands	30,000
0 thousands	0,000.

Breaking down the number and making a total for verification provides insight. The finished chart may be referred to in checking subsequent performances in reading and writing numbers.

A very different kind of chart is one worked out to indicate ways numbers are written and read for special business usage. This sort of chart would be worked out only after children had developed considerable facility with reading numbers. For gaining background information for making a chart to indicate correct reading of numbers in business relationships, children might be asked to investigate how telephone numbers, street numbers, license plate numbers, dates and the like are read. Experiences will differ widely. Some children will be ready to cite the difference in the way 1492 is read if it is a date, a telephone number, or, perhaps an amount of money. Other children will need time to collect data. When they have collected pertinent information, the teacher will guide their activities so that pupils become familiar with the variations in each category. For example, at least four types of telephone numbers are essential. Typical of the four are these: 5081, 6000, 7700, 2-2209. After each, the correct reading should be indicated.

Production of a time line can become another rich chart-making experience. If pupils work out a time line of their own, they discover relationships between fixed dates. With the complete chart for ready reference the scope and sequence of time become significant.

The direct teaching chart is another important type of chart. A simple illustration of this type is one titled *Things in Pairs*. For such a chart children will clip from periodicals pictures of items belonging in pairs and before mounting, sort the pictures into those in which the pair is two separate items and those in which the "twoness" is apparent but the two are fastened together as are a pair of eyeglasses. The term pair takes on meaning and the concept is extended through the activity. Other concepts such as that of instruments of measurement can be extended through the process of making simple picture charts.

Another illustration of the direct teaching chart is one designed to show square measure. On wrapping paper or building

paper of adequate size, children can measure off a square yard, the square feet in a square yard, and the square inches in a square foot, all in actual size. Relationships are apparent when they see the real thing instead of only the scale representation which appears in textbooks.

Not all charts will be teacher-pupil made. At times teachers will want to make up their own demonstration charts. The concept of average, for example, can be clarified by use of a few charts designed for the purpose. Such charts are effective teaching tools if used to stimulate thoughtful examination, followed by development of tentative generalizations which are experimentally tested.

Usually we teachers overlook numerous opportunities for pointing out the role number plays in other areas. Children will gain new understanding and appreciation if introduced to a chart showing fractions in music. They will be interested to discover that relationships between notes, and between the notes and rests in a measure are actually fractional relationships.

The few charts which it has been possible to discuss indicate that charts can be functional aids to the teaching of arithmetic. Each teacher will recognize many instances wherein the teacher-pupil developed chart will meet the needs of her particular group.

What is to be done with charts once their initial purpose is served? If placed in a flip-chart rack or in an oversize "book," the charts will serve as reference so long as they are needed. Also, they will serve as valuable review material and as milestones of progress. Since participation in the making of the charts is so important a part of the learning experience, it is obvious that most of the charts should be discarded at the end of the school year. Because they should be discarded it is neither necessary nor desirable that they be made of expensive, long-lasting material. Newsprint size 24″×36″, both plain and ruled, is inexpensive paper which proves satisfactory for most of the charts mentioned. Because it is large enough for material to be made visible from every point in the classroom, it is often preferable to the more expensive poster boards, usually available only in smaller size. Crayons and brush pens are both satisfactory for writing. Both are easy to write with and both provide easy-to-read markings.

EDITOR'S NOTE: There is much to be said in favor of charts especially when they are developed and made by pupils and teacher working together. As Miss Williams points out, at one stage a chart suggests that something is missing and hence may lead to a mode of discovery while at another stage a chart may be valuable as a summary or review device. Charts may serve many levels and many purposes. They are cheap, they need not be elegant, but they should be easily intelligible for the pupils.

An Important Announcement

Beginning with this issue THE ARITHMETIC TEACHER will be published six times each school year with issues in October, November, December, February, March, and April or May. Note the statement on the inside front cover of this issue giving details concerning subscriptions. THE ARITHMETIC TEACHER will continue to print articles dealing mainly with arithmetic in the elementary school. These will cover such materials as the results of experimental studies and researches, the practices that teachers have found valuable in the classroom, and discussions of content and method. The JOURNAL will be only as good as its content. The editors are constantly seeking articles that will help teachers in such a way that the youth of America may become more and more competent to understand, to think, and to perform in terms of the many arithmetical situations which confront the citizens in a democracy in which number, quantity, and form play an increasingly important role. As the famous Gauss wrote, "Mathematics is Queen of the Sciences and Arithmetic is Queen of Mathematics."

From time to time official business of THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS will be printed in THE ARITHMETIC TEACHER.

Casting Out Nines

Our Decimal Number System

LOUIS E. ULRICH, SR.
Principal, Sherman School, Milwaukee, Wis.

WE AMERICANS HATE DESPOTS—those who want us to obey, not to think. Or do we? Do those of us who turn the knob on our TV know why it works? Are despots always human? May we become slaves to mechanical devices without realizing it? What is the price we pay for the privilege of getting results without the toil of thought.

In the magazine, *Holiday*, for October, 1952 appeared an article entitled "Proof by Nine" in which the writer facetiously criticizes the mechanical, meaningless procedure employed to prove the correctness of multiplication and division examples. "Proof by Nine," or Casting Out 9's, as it is often called, is taught in Europe and appears in several American arithmetic texts. No one seems to know why it works, what it means, or what it proves. It is employed like this:

Rule: Add all the digits of a number, then take out all the 9's, if any, and you have a remainder. Multiply these remainders of the multiplicand and multiplier and again take out the 9's, if any, and you have another remainder. This remainder should be the same as the remainder of the total product after taking out the 9's, if the multiplication is correct. The principle involved is that equal numbers have like remainders after casting out 9's.

TO ILLUSTRATE: A

Multiplicand	76	7+6=13, 13−9=4
Multiplier	×68	6+8=14, 14−9=5
	608	
	456	
Product	5168	5+1+6+8=20, and 20−2×9=2

5×4 =20;
20−2×9=2

The remainders of both products check. Therefore, the multiplication is said to be correct.

Much of the confusion concerning the meaning of "Meaningful" arithmetic results from a failure to appreciate the processes involved. When these are understood, mechanical manipulations usually called "shortcuts" are discovered to be the culprits that hide meaning. It is one of the paradoxes of teaching that while educators strongly advocate meaningful arithmetic, we still have texts advocating the use of shortcuts without making any attempt to rationalize them.

Basic to a grasp of arithmetic principles and processes is an insight into the meaning of the decimal numbers and what they represent. A good approach then, to such an understanding, is to solve the riddle of "Proof By Nine." Why does it work? What does it prove? What does it mean?

Casting Out Nine on the Abacus

Some primitive tribes and most ancient civilizations employed some form of counting or tally device such as an abacus. Many of these devices seem to have a relationship to the fingers and hence also employ the base ten of our number system. Legend suggests that a primitive chieftain in counting and recording the number of sheep in a fold, placed a pebble

on the ground in a spot corresponding to the *one's* place on the abacus as each sheep passed by. When he had put as many pebbles in the *one's* place as he had fingers on both hands, he took the ten pebbles out of the *one's* place, placed one of them in a second spot corresponding to the *ten's* place, and put back in his bag or on a pile the other nine pebbles. That is, he *cast out* nine pebbles. He had one *remaining* pebble in a new position. Many teachers show this process on a "10-bead abacus," with "number sticks" or similar device when they teach numbers and *place value.*

This process of "pebble-tally-counting-recording" may be continued indefinitely by repeated *casting out* of *nine* and moving a pebble into the next higher order. When the chieftain had counted 38 sheep, his record showed 3 pebbles in the *ten's* place and 8 pebbles in the *one's* place. In doing this he had taken 30 pebbles out of the *one's* place, had *cast out* 3×9, and the 3 pebbles now in *ten's* place correspond exactly with the number of nine's thus far cast out.

Had the chieftain counted the sheep and put all the pebbles in the *one's* place without changing to tens there would have been 38 pebbles in the *one's* place.

If he had later taken out the 9's from the 38 the remainder would have been 2 pebbles. By putting 3 pebbles in the ten's place and leaving 8 pebbles in the one's place, 9 pebbles were cast out of only 27 pebbles and not of the whole 38. Therefore 9 pebbles must be taken out of the 11 pebbles remaining on the abacus so that the 9's are removed from the whole of 38. This leaves a remainder of 2 pebbles and explains 38, 11, 2. ($38 \div 9 = 4$, with *2 remaining.*)

Suppose we now analyze the number 5168, which is the product in illustration A. On the abacus it may be represented as:

M	*C*	*X*	*I*
ooooo	o	oooooo	oooooooo

5168 uses 20 pebbles

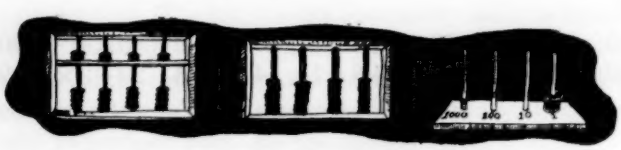
How did these counters get there? 5168 was built up as follows:

a. 5168 pebbles or counters were put into the *one's* place one at a time.
b. In changing to *tens,* 516 9's were consecutively cast out of the *one's* place and 516 put into the *ten's* place leaving 8 remaining in the *one's* place.
c. In changing to *hundreds,* 51 9's were consecutively cast out of the *ten's* place, 6 retained in the *ten's* place, and 51 put in the *hundred's* place.
d. In changing to *thousands,* 5 9's were consecutively cast out of the *hundred's* place, 1 retained in the *hundred's* place, 5 put into the *thousand's* place.

The 20 pebbles in their respective places now represent the number 5168. But these 20 pebbles physically contain 2 nines. After these are cast out there are but 2 pebbles left. This explains 5168, 20, 2. In all 574 nines were cast out leaving 2 pebbles on the abacus.

There is of course a simpler way to cast out nines to find the remainder. Merely divide a number by 9, as: $5168 \div 9 = 574$, *2 remaining.*

All this simply means that equal numbers have identical remainders whether we cast out nines before or after total computation.



FORMS OF THE ABACUS

Casting Out Nines in Processes

Since addends produce their sum, the addends together, after casting out nines, should have the same remainder as the sum, or the addition was faulty. To illustrate:

Addends	256 − 13 − 4	
	875 − 20 − 2	
	678 − 21 − 3	
Sum	1809 − 54 − 9 − 0	1809 − 18 − 9 − 0

In subtraction the remainders of the subtrahend and difference together equal the remainder of the minuend, when the subtraction was correct.

Minuend	1809 −	18 − 9	−0
Subtrahend	−678 −	21 − 3	−9 − 0
Difference	1131 −	6 − 6	
		— −	
		27 − 9	−0

In multiplication, the remainder in the product of the remainders of the multiplicand and multiplier, equals the remainder of the total product. (See illustration on page 77.)

In division the product of the remainders of the quotient and divisor plus the remainder after division, equals the remainder of the dividend, when the division is correct.

68		
76)5170	76 − 13 − 4	4
456	68 − 14 − 5	×5
610	5170 − 13 − 4	20 − 2 + 2 = 4
608		
2		

Number, therefore, is interesting in itself when we see how it is structured. The reader will now see that 10, for example, does not represent ooooooooo individual discrete objects, but rather one *ten* and no *ones*. Ten represents the mental processes by which we transform each ten smaller units to one larger one. Thirty-eight for example is made up of 11 decimal units—three *tens* and eight *ones*. It does not symbolize:

ooooooooooooooooooooooooooooooooooo,

which is an aggregation of ones too numberous for the mind to visualize. Hence, we mentally change thirty of the thirty-eight ones to fewer larger units more easily visualized as 3 tens.

Analysis shows us that casting out nines and carrying ones to higher value decimal places describes what was actually done with concrete materials and devices in a primitive social situation. It does not, however, describe what we now do mentally and on paper to solve a social situation involving numbers.[1] Mentally 38 means 3 tens and 8 ones, 11 decimal units in all, to represent in value thirty-eight individual ones.

Concretely and tangibly 38 means 30 pebbles taken out of the *one's* place, 3 put to the *ten's* place, and 8 left in the *one's* place. $38 = 3 \times 9 + 3 + 8$.

Our current arithmetical computation is no longer the manipulation of things or tangible devices, but it is thinking about things in terms of transforming them to fewer larger decimal units. Our numbers now represent what we think decimally about, and not what we do physically with, the things we sense.

Proof by nine or casting out nines is difficult to understand arithmetically because it does not represent modern computation. It is descriptive of primitive and ancient calculation with tangible devices like the various forms of the abacus. It may be done mechanically without meaning like punching the keys of a comptometer. In closing it is interesting to note that the Latin word for pebble is calculus. The ancients calculated, we abstract.

EDITOR'S NOTE: Does Mr. Ulrich's illustration of "casting out nine" in primitive forms of reckoning constitute a proof for "why it works" in our current algorisms of computation? Should we teach this as a standard method of checking? Has the process of "checking by nines" any real value in itself in understanding numbers and processes or in practice in using numbers? The editor has long used this check with certain groups of pupils and found it intriguing and worthwhile.

[1] Ulrich, Louis E., Sr. "A Study of the Process of Transformation," by Louis E. Ulrich, Sr. *The Mathematics Teacher*, November 1947, p. 346 ff.

The Decimal Is More than a Dot

FRANK C. ARNOLD
Principal P.S. 148, New York City

MISSOURI MUST BE THE BIRTHPLACE of all teachers since the constant cry of each is "Show Me." We in District 45 and 46 under the aegis of Assistant Superintendent Max Gewirtz attempted to do just this in our training program for newly appointed teachers. (Districts 45 and 46 are part of the New York City school system. There are 37 elementary and junior high schools with approximately 25,000 pupils and 1,100 teachers.) The demonstration teacher in each case was helped by the Elementary Mathematics Coordinators. The following is an account of one of the mathematics lessons which the neophytes of the 5th and 6th grades viewed.

Parenthetically the weakness of this viewing process, so prized by teachers, is that the new teacher does not have enough background to be able to know the graded steps that precede and follow the lessons she views. For this reason, in our demonstrations the viewer received a "Before-and-After" outline which was discussed at the briefing and follow-up periods. The outline of the lesson which we are describing will be found at the end of the article.

A Lesson on Decimals

The lesson chosen for demonstration in this instance was concerned with the early development of decimals. The reason for this selection was that decimals are so deceptively "easy."

Now to the development of the lesson—

Actually preparation for this lesson in decimals began "many grades ago" with the emphasis on place value and on fractions. However, not to burden the reader with the sequential development through the years, we shall go back only a short time before the lesson itself.

During this preparatory period the class explored instruments which record measures in tenths. The teacher took the class to visit the neighboring gasoline stations to examine the calcometers and to observe how they record the amount of gasoline sold and its cost. Those children whose parents own cars were asked to observe the workings of the odometer. To be sure that each child had such opportunity, the teacher took the children in small groups on short rides. During the ride he directed their attention to the way the odometer recorded the distance traveled. One morning a pupil whose father has an auto supply store brought in an odometer. By using a small motor the children learned how mileage is recorded. Another pupil demonstrated the use of the cyclometer on his bicycle. Clinical thermometers were examined. Large scale models of an odometer and a clinical thermometer were constructed. Movable strips were used so that the children could "work" the odometer model. Of course, in addition to the preparation outlined above, the place value of whole numbers and the ratios involved in place value were stressed throughout the term.

The lesson itself began with the recall of the visits to the gas station, the manipulation of the odometer etc.

The children discussed the various ways in which quantities smaller than one whole had been recorded on the various instruments they had observed. i.e. as a common fraction on the calcometer; as a differently colored number to the right of the whole numbers on the odometer, etc. They demonstrated on the model of the odometer how tenths of a mile are recorded; how ten tenths "automatically" become one more mile.

The teacher then showed whole miles

on the odometer and asked such questions as, "What kind of numbers are these?" "Whole numbers." The teacher showed parts of a mile on the odometer and asked the children to identify this type of number. Some of the responses were—"smaller than a whole number," "smaller than a mile," "a fraction." On the blackboard the children recorded in fraction form (1/10, 2/10 etc.) what was shown on the odometer.

Then the teacher asked the children to represent 1 mile by a line on the blackboard. He directed them to represent one tenth of a mile on this line. They discussed the ratio between 1/10 of a mile and one whole mile. Two tenths were shown on the line. The child was asked to show 2/10 of a mile on the odometer. At this point the teacher went back to review the meaning of the place value of whole numbers. The teacher showed the units on the abacus. "If you were at the gas station getting gas, what would this mean?" "One Gallon of gas." "If you were riding in a car what would this mean?" "One mile." A child was asked to represent on a chart the unit seen on the abacus. After discussion children were asked to show 10 miles and 100 miles on the abacus. The numbers were also written on the chart. Now using the chart below the children made the following comparisons and generalizations.

Tens	Units	Tenths
1	1	1

The "1" in the tens place has ten times the value of the "1" in the units place; the "1" in the hundreds place has ten times the value of the "1" in the tens place etc.

The children then compared the value of the "1" in the tens place with that of the "1" in the hundreds place; the "1" in the units place with the "1" in the tens place and made the following generalizations. The "1" in the tens place has one tenth the value of the "1" in the hundreds place: the "1" in the units place has one tenth the value of the "1" in the tens place.

Once more attention was drawn to the odometer. The teacher asked a child to show one tenth of a mile on the odometer. "Can we show this on our abacus?" was the teacher's question. After some discussion as to the position, the number of beads needed etc. two children added the needed column of beads to the right of the units column. It is interesting to note that the children had no difficulty in labelling this column tenths. If children do not have an understanding of the ratios involved in place value, the usual label given to this column is halves.

"We have shown one tenth on the odometer, on the line diagram, on the abacus. Now let us show it on the chart." Here, too, the children found little difficulty in writing and labelling the column to the right of the units place on the chart.

When this was done the teacher continued, "When we write numbers, do we always label them? Suppose we did not have the labels, how would we know how to read this mileage?" The teacher now wrote 11 without labels. "What mileage does this say?" "Eleven miles." "Look at the odometer. What do we want it to say?" "One and one tenth miles." "How shall we write this so we can recognize it as one and one tenth miles?"

This was the $64 question that "stumped" the class. The teacher returned to the odometer. "How was 1/10 of a mile shown on the odometer in the car?" "It was shown by a different color." Stimulated by this reminder, the children went on to give suggestions such as, "Put in a box around the number," "Make the number smaller in size," "Make the number another color" etc. Some of these were ruled out as not practicable or clumsy. Finally a child suggested, "When you have cents, which are less than a dollar you use a dot." The teacher affirmed this

child's suggestion but brought out that this form was not universal. The form $\frac{\text{units}}{1}\,.\,\frac{\text{tenths}}{1}$ was written on the board, and after discussion, the columns were labelled.

The children showed one tenth in a variety of ways including the decimal form.

Then the teacher gave practice in showing, one tenth, two tenths etc. in all forms —on the odometer, on the abacus, on the chart, as the common fraction, as a decimal fraction.

As a homework assignment one group was asked to find out how tenths are written in different countries other than the United States. The other groups were asked to find instances in which tenths are written in decimal form in newspapers, in social studies books, in railroad schedules etc.

We feel that after this lesson the young teachers realized that there was more to teaching decimals than the dot.

Outline for the Lesson

PRELEARNINGS

- Understanding of place value of whole numbers
- Understanding of common fractions
- Understanding of relationships between and among common fractions and whole numbers
- Special emphasis upon the tenth as a common fraction
- Understanding of and generalizations concerning ratio of tens to units, hundreds to tens, etc., and vice versa

AIM OF THE LESSON

- To develop the concept of a decimal fraction (initial development)

THE LESSON CONTINUED

- Further emphasis on developing the concept of the tenth as a decimal fraction
- Representing, reading and writing tenths as decimal fractions
- Introducing terms *decimal fraction* and *common fraction*
- Emphasizing common fraction equivalents for decimal fractions and vice versa
- Adding and subtracting tenths (decimals and mixed decimals involving tenths)
- Developing concept of cents as part of a dollar
- Developing the meaning of hundredths as decimal fractions
- Emphasizing relationships among hundredths and the whole, hundredths and tenths, etc. (place value)
- Adding and subtracting hundredths (decimals and mixed decimals involving hundredths)
- Adding and subtracting tenths and hundredths (decimals and mixed decimals)
- Generalizations concerning the denominators of common and decimal fractions
- Multiplying and dividing decimal fractions by whole numbers
- Similar procedure with thousandths (for more mature children)

EDITOR'S NOTE. It must be a tremendous job in a large city to induct and indoctrinate young teachers into a program of developmental arithmetic in which understanding and meaning and significance replace the older pattern of learning computations. It is sad to note that teacher training programs apparently do not do a satisfactory job in really educating teachers in the necessary concepts, ideas, and principles of the work they are to teach. Many teachers who read Mr. Arnold's article and who live in smaller communities may feel that their pupils have a more varied and richer experience outside of school than is open to boys and girls in the large cities.

American Education Week

November 6–12

It is a good school policy to call to the attention of the general public the importance of education to the citizens of our country. Many parents are reached through report cards and parents' organizations but a sizeable group of citizen taxpayers do not have this contact with the schools. The National Education Association through its sponsorship of American Education Week has done much to stimulate interest in learning. Schools and teachers must however take the responsibility for localizing the movement. Packets of materials for promoting American Education Week are available at low cost from the N.E.A., 1201 Sixteenth St. N.W., Washington 6, D.C.

Developing Understanding Through Counting

E. C. Blom
State Teachers College, Fredonia, New York

Because a learner's first impression tends to be permanent, some apparently elementary conventions are actually very important. Many factors affect the learning of mathematical concepts. It is easy to make the mistake of assuming that beginners learn easily and quickly the many meanings of terms, symbols, and procedures, and their respective uses that have become automatic for an adult. We should, therefore, place great emphasis upon arithmetic instruction in the early school years.

Children often have rather well developed number concepts before they begin formal study. Unless new concepts coordinate readily with those already established, we may find a most difficult learning situation. In fact, it may be far harder than if no concepts had been established. While a number of approaches to developing a concept are possible, the teacher must choose the particular one suited to the learners' background.

For instance, some children learn to count in an abstract fashion before they go to school. They can say the names of numbers in a desired order far beyond what one would expect, and yet not recognize the possibility of applying these abstract terms in activities familiar to them in their associations with their parents, children, and even their pets or playthings. Simply because a little girl is able to say "five" and then "six" gives one no assurance that she will associate these words properly when observing two groupings of five eggs and six eggs. She may not even realize that the larger group has only one more egg than the smaller one, or that there is any numerical difference between the groups. On the other hand it is not unusual to find children who know for instance, that two pennies and seven pennies total nine pennies, yet are not readily able to recognize that two and seven of anything would amount to nine of these things, or finally that two and seven are nine. Some children come to our classes with a background of rich associations of quantity concepts, while others might have very few.

If an individual has learned to subtract seven from nine by taking seven away from nine he might not realize that the difference is the quantity which when added to the subtrahend gives the minuend. The latter concept is very helpful in understanding subtraction where signed numbers are used, and is the basis for the additive method of subtracting. Children who have had experience making change with money might find the additive method of subtracting a sensible procedure. It is the writer's belief that adults use the additive method for subtraction more frequently than they realize.

The Role of Counting

Some of the earliest contacts with quantitative matters arise in counting. Although each child may begin by matching one thing with another to get a one-to-one relationship, as in determining if there is a cooky for each person present. Different children may have different ways of interpreting and expressing relationship. It is advisable, then, that the teacher diversify her approach sufficiently to utilize the pupils' varied backgrounds. Most small children participate in many kinds of parties where they learn about servings, seating arrangements at tables, and other ways of grouping things and persons. Games like dominoes, lotto, bingo, monopoly, and such team games as basketball,

baseball, football, hockey, and tennis are rich in similar experiences. Many children participate in actual business ventures like selling newspapers, operating lemonade stands, and helping their parents in stores, or by selling vegetables from the family garden.

Names for numbers, particularly the commonly used smaller ones, are almost associated with something concrete. Very large numbers and very small fractional parts which are associated with situations not readily imagined tend to seem abstract, and should not be considered in the early stages of learning. Even with smaller numbers, the primary teacher should provide many and varied initial associations with objects.

The very fact that a learner tries to reason can get him into difficulty where apparently similar situations are handled quite differently. Suppose a child is asked to count the digits of his hand. He finds that it makes no difference whether he starts with the thumb, the little finger, or any other finger; the total is always five. Now he has often heard a particular finger identified as the first finger, another as the middle finger. Here he finds a different kind of number—one associated with arrangement. He is in trouble if he begins counting with the little finger because what is customarily called the first finger becomes the fourth; and there is no middle finger as such.

It is not unusual to stress the fact that numbers in arithmetic as written are sums valued according to positions occupied by the digits. Thus, 325 means three hundred plus twenty plus five. When the student, in a higher grade, sees letters written in adjoining positions he must understand that a product is indicated. Here *prt* means *p* times *r* times *t*, the relative position of the adjoining letters makes no difference in the product.

Counting is an important feature of many quantitative aspects of living. The pupil should be encouraged to use varied ways of counting so long as he understands that what he is doing amounts to matching one at a time. The abacus, simple adding machines, speedometer, packing cases for eggs, and other devices can be used for enrichment and for physical participation on the part of the pupils. For young children particularly, physical activity can help to make classwork delightful and meaningful.

Drill produces results largely in proportion to the stress placed upon discovering and understanding facts. Simply memorizing facts (combinations, for instance) usually results in narrowly limited values. Ability and readiness to work with numbers is enhanced by rich firsthand experience. Naturally, the wide background of the teacher must be utilized to help the pupils discover interesting and important applications.

Although relatively few combinations result in numbers from one to nine, the basic principles of the four fundamental processes can be established rather effectively within this limited range. Pupils can easily manipulate, draw, or imagine a small number of things, especially if each operation is seen as a kind of counting.

In addition and subtraction, the one-to-one relationship is retained. Thus two individual things combined with (added to) three individual things produces a combination (equals) of five things. Counting is all that one actually needs to do to carry out the operation. After the learner understands thoroughly how new groupings (sums) are obtained, it is soon enough to develop the idea of combinations and to stress the time saving advantage of memorizing them. It seems unreasonable to force memorizing combinations before the learner sees a real reason for doing so.

Likewise, such symbols as $\times$, $+$, $-$, $\div$, and $=$ should be introduced as soon as the learner realizes the convenience resulting from using them. However, the use of symbols should be incidental to thorough understanding of the processes or relationships they represent, so that wrong interpretations are at least minimized. Some

children are not sure which number is to be divided by the other in a setup like $5 \div 8$. Then too, it might be very helpful in the study of multiplication of fractions if the expression "$\frac{1}{2}$ times $\frac{1}{4}$" is written as "$\frac{1}{2}$ of $\frac{1}{4}$." Some authors recommend using the symbol $-$ to identify negative numbers only, never as an indicator for the subtraction operation. Certainly, working with symbols of almost any kind can be very confusing unless backed by thorough understanding to the extent that the symbolization can be expressed in the learner's own words.

The teacher in illustrating the take-away process in subtaction may ask five of her pupils to stand in a group and then have two of them leave. Counting would be required twice. By a third counting, the number of pupils remaining can be ascertained. The terms remainder and difference can be explained at this point. In this case the remainder or difference is three children, that is, the number left after the operation has been performed.

To give another interpretation for difference, the children may count to see how many must be added to make a new group of five. Here the pupil will see that the remainder or difference (three children) is the number which must be added to the amount subtracted (two children) to obtain the original number (five children) upon which the subtraction was performed. That is, the difference (three) plus the subtrahend (two) must be equal to the minuend (five).

Counting is also the basis for multiplication and division. However, these processes utilize groups of more than one, whereas addition and subtraction use one-to-one relationships. In learning the value of three *two's*, the children may actually manipulate two groups of three items or three groups of two items each. They are counting by *three's* or by *two's*. They are adding over and over again a definite quantity; hence, multiplication is repeated addition.

If two is multiplied by three, the child thinks "three of the *two's*," and accordingly groups whatever he is working with as (two plus two plus two). However, he could arrange the groups as (three plus three), where he would have three multiplied by two. The result (product) is the same for both arrangements. For the first arrangement the multiplier is three and the multiplicand is two. The second arrangement has two for the multiplier and three for the multiplicand. The addition idea for multiplication could be helpful for the child in situations like where he remembers that five times nine equals forty-five, but cannot recall the product of six times nine. Here he simply adds another nine to the forty-five obtaining the product fifty-four.

Likewise, division is repeated subtraction in groups of more than one. While thinking of nine divided by two we can proceed by removing from nine objects groups of two each until all such complete groups are removed and then count the number of groups. Our entire operation results in removing four and one-half complete groups. It is easy to see that the division process gives, as the quotient, a quantity which is the dividend blocked off into groups, each of which is the size of the divisor. This same division problem might be thought of as two children sharing nine cookies. Equal shares would result when each child has obtained four and a half cookies.

A mathematics laboratory well equipped with objects, pictures, simple machines, tools, and necessary supplies constantly available for use by the pupils can keep mathematics from seeming abstract. Too, a thorough understanding of mathematical procedures can be established. Children learn best by doing.

EDITOR'S NOTE: Dr. Blom has used a number of illustrations to show the role of counting in the development of understanding. He has called attention to the need for working with real things for early associations of number and counting and that the possession of a vocabulary of such a simple item as "five, six" does not necessarily mean that the child possesses a good concept of number.

Measurements

A Skit by Eighth Graders and Their Teacher

LORENA HOLDER
Dallas, Texas

Shirley: We have studied many kinds of measures in the sixth and seventh grades. We have learned to add, subtract, multiply, and divide these numbers, calling them denominate numbers because they all have names. Sometimes we call them compound denominate numbers because we often use several numbers with different names to represent one value. In our work with other kinds of problems, and in our everyday experiences we seem to find more use for long, square, and cubic measures than any of the others, so we decided we would like to know more about these measures. We began by finding out how measures of length came into use. Tommy, will you tell us about some of these strange old ways of measuring?

Tommy: Suppose for a few minutes you were a prehistoric man and that you wished to make a bow like the one your neighbor has. Your neighbor cannot do without his bow while you copy it; therefore, you must measure it by comparing it with something else. It occurs to you that your arm from the point of your elbow to the tip of your middle finger will make a good measuring rod. You put your forearm down on the bow and find that you cannot measure it exactly with that measuring rod only, because the bow is not an even number of forearm lengths long. It then occurs to you to spread out your hand. The span from the end of your thumb to the tip of your little finger makes another good measure. You find that the bow is somewhat longer, however, than two forearm lengths and one thumb-to-little-finger span. You still need a measure which is a little shorter. Now you use the width of your middle fingers. In the end, you find that the bow which you wish to copy is two forearm lengths, one thumb-to-little-finger span, one fore-finger breadth, and one middle finger breadth. Having thus measured your neighbor's bow, you can make your own practically the same length. In this measuring you have used the *cubit*, the *span*, the *palm*, and the *digit*. We find all these measures in Bible times. (Tommy measures a bow on the table as he talks, and in his summary he shows the measures on his arms and hands.)

Bill: (Spontaneously) That isn't much worse than young Brown who got a job in a shipyard. The first morning he was there, the foreman gave him a two-foot rule and told him to go measure a large steel plate. Brown returned in twenty minutes. "Well," inquires the foreman, "what's the size of the plate?" The youth displayed a satisfied grin and answered, "It's the length of this rule and two thumbs over, with this brick, and the breadth of my hand, and my forearm from here to there, bar the fingernails." (Bill displays a two-foot rule, measures his thumbs together, picks up a brick, measures his arm from finger tip to elbow.)

Wanda: (Spontaneously) And it's no worse than Miss Allcott who worked in a war production plant and thought that a ten-thousandth of an inch was the same as a tweensy-weensy little bit.

Shirley: As civilization advanced there developed more need for definite units of measurement, so we have the story of some English measures of length. This is chiefly the story of attempts to

have the same measures used throughout England, and to have these measures exactly the same wherever they are used. Loretta, will you tell us about some early English measures?

Loretta: (Displays picture) In the fifteenth century the English Parliament started with the *inch* in standardizing the measures. An inch was determined by laying three grains of barley corn end to end. The grains were to be dried, and were to be taken from the middle of the ear.

(Demonstrates with a long piece of tape) King Henry I is said to have started with the *yard* in an attempt to standardize measurements in his time. He decided that the standard yard was to be the distance from the tip of his nose to the end of his thumb.

Sharon: (Spontaneously) Little Betty's grandmother used this old fashioned method of measuring a yard by stretching the goods at arm's length, holding one end of it up to her nose. One day Betty came up to her grandmother with a piece of ribbon and made this strange request, "Smell this, grandmother, and see how long it is."

Nelda: The wit of the Irish is shown in this little episode. The Irishman's eyes twinkled with mischief as he entered a shop where there was a notice displayed saying that nearly everything there was sold by the yard.

"I'll take a yard of milk," he said to the storekeeper. The storekeeper wrinkled his brow for a moment and then dipped his finger in a bowl of milk and drew a line a yard long on the counter. By this time several people had come in and were watching the proceedings with keen interest, wondering what Mike would do. Without moving a muscle, the Irishman asked the price.

"Five cents," said the storekeeper confidently.

"All right, my boy, roll it up and I'll take it."

Shirley: Since most of the early American settlers were from England, it is not surprising that the standard measures of length in use in America were brought over from England. Leda, do you know any other unit of length?

Leda: Yes, in 1790, during the French Revolution, the National Assembly appointed a Committee of the Academy of Science to study the matter of finding a suitable system of weights and measures. This committee worked out an entirely new system which is called the *Metric System.* It is named from the "meter" which is one ten-millionth of the distance from the earth's equator to the North Pole. The meter is 3.37 inches longer than our yard, which is 36 inches. Although the metric system is not in general use in our country, it has been a legal measure here for more than 60 years. Our boys who went overseas had to learn to use this system of measure. There is much in favor of our adopting this sytem to conform to the systems of other countries, as an aid to foreign trade. The meter corresponds to our yard, and the kilometer to our mile.

Shirley: What is our measure for long distance?

Wynona: The mile.

Shirley: What is a mile?

Wynona: Mathematicians say it is 5,280 feet, but this answer was not enough for two theatre owners who were recently in litigation in the Supreme Court. One claimed that the other had entered into an agreement not to open a moving picture house within a mile of his own. This raised the legal question, "What is a mile?". One lawyer argued that the distance should be measured "as a crow flies." The other lawyer argued, "it should be measured as a man walks on the public highway." It was up to the judge to decide the question. He decided that since MEN, not crows, attended the motion picture shows, the mile should be measured as a man walks on the highway.

Jack: How long is a statute mile?
Kenneth: 5,280 feet.
Jack: What is a nautical mile?
Kenneth: A unit of measure of distance on the sea.
Jack: How long is a nautical mile?
Kenneth: 6,080.2 feet.
Jack: Why is a nautical mile longer than a statute mile?
Kenneth: Probably because things swell in water.
Kenneth: Do you know why the school yard is always larger at recess?
Jack: No, why?
Kenneth: Because there are more feet in it.
Tommy: From these questions and answers we can get some idea of the need for definite units of measurement; and *what*, *when*, and *how* we need to measure.

An endless variety of problems arise in various occupations which involve practical measurements of lines, planes, and solids. We are not concerned with problems today, but we should like to know the names of the various forms and measures we use every day. By drawing lines and angles in various lengths and positions on plane surfaces we form what we call plane figures. These figures have names, just as you and I have, and we must call them by their names when we talk about them. Also, many interesting problems involving space relationships arise which may be solved through a knowledge of the laws of similar figures and indirect measurement. For example: computing the height of a tree or a flag pole from the lengths of their shadows, or by sighting the top of an object along the hypotenuse of a right triangle. These sorts of problems show us the need for the study of geometry, since geometry is that branch of mathematics that treats of measurements of lines, angles, planes, and solids, with their various relationships. Plato, the famous philosopher, thought the study of geometry so important that he had inscribed over the entrance of his school, "Let none ignorant of geometry enter here." We should like to tell you some of the interesting things we have learned about these measurements. First, we study lines. (Displays placard with various kinds of lines drawn on it)

Brenda: A line is a continuous extension in any direction, purely imaginary, without breadth, thickness, or end. When I look at you there is a line from my eye to your eye, but we can neither see nor touch this line. Another interesting thing is that the line does not stop with our eyes but goes on continuously and indefinitely in both directions. When we draw a line on our paper to represent a line, we must use a well sharpened pencil so the mark will be as fine as possible, since a line has no width. Lines are vertical, horizontal, oblique or slant, parallel, perpendicular, broken, or curved. Many variations of lines are used in different fields; such as: Art, Arithmetic, Geometry, Daily Papers, Household Work, Play, Building, Business, etc.

How wide is a line, no one can tell;
It has no width, as you know full well.
The only measure a line can boast
Is the one we know as *length*.
A line can measure how *far* and how *long*,
But it never can tell how *thick* it has gone.

Bertram: Lorrie Brooks in *The Dallas Morning News* says a line is the only thing that can go in opposite directions at the same time.
Joker: I know another definition for a line!
Brenda: What is it?
Jack: A line is a unit of measure .008 of an inch.
Brenda: Where did you learn that?
Jack: From Mr. Ripley.
James: A *point* is the intersection of two lines. Its purpose is to indicate position.
Nelda: (Holding drawing to illustrate)

An *angle* is the opening between two lines.

This opening is not fixed by the length of the lines.
As the opening changes in number of degrees
The name indicates the amount of the fees
Until a complete circle has been made to please
And thus formed six kinds of angles,
To unscramble the tangles.

Leda: (Holds up blank sheet of paper) *Surface* has two dimensions, length and width. It has no thickness; therefore we cannot pick up a surface without picking up the object. There are many illustrations of surface: the surface of the earth, the water, the floor, the walls, etc. Surface may be regular or irregular. In our drawings we use a regular, smooth surface which we call a plane surface.

Tommy: (Showing constructed triangles)

The fewest lines that can be drawn
To hold a structure against the storm
Are three, forming a triangle.
(*Show picture of a bridge showing triangular frame work*)
"I am Sir Triangle
Of strength and might
I support a bridge
And hold it tight."

Jack: What is the RIGHT angle from which to approach a problem?

Tommy: The right angle from which to approach a problem is the TRY angle.

Bill: (Holds large quadrilateral upon which the other forms are mounted)

"We are a family large and fair
Live without us if you dare!
We make your houses straight and true
We are the windows you see through
And if you swing on the garden gate
Our lines are true, but you have changed our shape.
Here is the family merry and sad
Quadrilateral is the great grand dad.
Trapezium, Parallelogram, Rhomboid, Triangle, Rectangle, Square
You meet us here and you meet us there.
We go to school, we stay at home,
And in the wide, wide world we roam.
Whatever our name, whether great or small,
Four sides and four angles have we all."

James: (Holds placard with various polygons mounted)

Now on the surface plane
With lines and angles we may draw with much gain.
With lines the same length and angles equal
We may draw symmetrical figures
Which we call regular polygons.
"We are the Peers of the Polygons
We are the true and noble ones
Our sides are equal and angles too,
And we do as we are supposed to do,
For we have laws we all obey,
Sunday, Monday, and every day."

Wanda: (Poster shows pentagon and star)

"I'm the five sided pentagon;
In the distant blue of the sky afar,
Shines my cousin, the five pointed star."

Loretta: (Poster shows hexagonal figures and snowflakes)

"I am the useful hexagon
Come with me and have some fun
I am wily, shy, and full of design,
To the Ancients I was a mystic sign,
To the circle I am closely akin,
An equilateral triangle fits within."

Jack: (Placard shows nonagon)

"I am the stately nonagon
A proud nine sided polygon
A useful life is not for me
I'm merely here for all to see."

Patsy: (Placard with various polygons mounted)

"There is no end to polygons
Sisters, cousins, uncles, sons,
But we must meet upon a plane
And clasp our hands to meet again,
We must be straight and never curve,
From these three truths we may never swerve
And we will be good polygons."

Joker: What geometric figure tells us that the parrot is lost?

Patsy: Poly-gon.

Brenda: (Placard shows a circle and the symbol for *pi* in an upper corner)

"I am the god of the Polygons
Try as they may they can never be one
I'm round and rolly and full of *pi*,
I live in the earth and in the sky."

INSTRUMENTS

Bill: (Instruments displayed on table, Bill shows each as he names it) For our work with lines and angles and all these figures we must have special drawing instruments besides our finely sharpened pencils. A ruler, or rule, is an instrument with which we draw straight lines and by means of which we measure the length of lines and distances. (A 6-in. rule; a 12-in. rule; a 2-ft. rule; 15-in. rule; 3-ft. rule or yard stick; z folding carpenter's rule; a foot rule divided into 10ths and 100ths; a tape measure used by tailors and dressmakers; T-square for drawing parallel and perpendicular lines; a fisherman's rule—and would you believe it? It will measure a fish 26 inches long while the rule itself is only 6 inches long!)

Kenneth: A protractor is the instrument with which we measure the opening of an angle.

Now that we are ready to start
I know we shall be very smart
To make this protractor come in handy
For measuring degrees it is a dandy!

(Small protractor measures 180 degrees of ½ circle.

(Round protractor measures 360 degrees or a whole circle.

(Large protractor to use on black board or large drawings.

Bertram: The compasses are for drawing and dividing circles or transferring measurements. The small ones are used to draw on paper, the large ones to draw on the black board or large drawings.

Tommy: We have enjoyed meeting with you teachers on this occasion, and we thank you for your kind attention.

EDITOR'S NOTE: This skit on measurements was prepared by the pupils in the eighth grade and was presented at the luncheon meeting of the Texas Council of Teachers of Mathematics. Miss Holder is president of the Texas Council. Copies of the skit may be obtained by writing to her at 312 West 9th St., Dallas 8. Children delight in drama and hence a skit such as this serves as a stimulus. The greatest value probably lies in the reference work involved in preparation. The role of language and of writing should not be minimized.

Review of "IMOUT"

The game device IMOUT (I'm out) is an adaptation of the popular pastime of bingo to addition and subtraction combinations. Cards with twenty number combinations are furnished to each child who then draws at random or, by use of spinner, is furnished an answer which he fits into one of the squares on his card. When he has completed a row or column on his card of combinations he calls out "IMOUT." The device has the value of combining the features of a game with drill in abstract arithmetic. Of course the teacher must check on the accuracy of the child's work. As with all supplementary drill and learning devices, much of the value depends upon how the item is used. Here is an item which may be very useful for drill on a restricted number of combinations. Certainly it will stimulate interest among pupils. It has the advantage of not applying time pressure. It has a weakness in that many pupils may have very little to do during the operation of the game. The device is available at $3.60 postpaid for the set of 48 cards from The Educational Aids Co., 460 The Arcade Bldg., Cleveland 14, Ohio.

Arithmetic Teacher Index

An index listing titles and authors of articles published in THE ARITHMETIC TEACHER in 1954 and 1955 will be printed in the issue for December, 1955. The items for 1954 will be printed as a unit so that those who wish to use this index for bound volumes may do so. Likewise the 1955 content will form a unit. Thus 1954 and 1955 numbers may be bound as one unit or as two separate units.

=, Equal or Equals?

LEE EMERSON BOYER
State Teachers College, Millersville, Pennsylvania

SOME TIME AGO a group of our students were planning an arithmetic television program. They raised the question whether $3+2=5$ should be read "Three plus two *equals* five" or "Three plus two *equal* five." Also, they asked, "Does it make any difference if we speak about concrete objects?" For example, should we say, "Three bears plus two bears *equals* five bears" or "Three bears plus two bears *equal* five bears?"

At the moment the writer said he could state his opinion but he would prefer to help "find the answer by consulting the 'authorities'." But we found the "authorities" on this subject were not easily located.[1] Indeed, most textbooks were extremely vague if not just plainly non-informative. Sometimes authors gave the impression that = could be read either "equals" or "equal"!

To pursue this question further we attempted to learn how in-service teachers felt about the issue. We did this by giving them a list of statements in which the = part was always spelled out. We then asked them to place a *c*, for correct, or an *i*, for incorrect, after each statement according to their opinion of correct English usage. Following this they compared their markings with our key which appeared on the back of the sheet. The reactions were almost as vague as the suggestions of the textbook-writer authorities. In every case, however, the teachers, as groups, showed marked interest in the problem and indicated a desire to "know the rule!"

The list of eighteen statements is reproduced here as an opportunity for the reader to check himself. The list is drawn up so that fifty per cent of the statements are correct, fifty per cent incorrect.

1. 5+2 equal 7.
2. 7−6 equals 1.
3. 2 multiplied by 3 equals 6.
4. 8÷4 equal 2.
5. 71 times 486 equals 34,506.
6. 2×6 equals 12.
7. 12 multiplied by 2 equal 24.
8. 5 bears plus 4 bears equal 9 bears.
9. 1 bear plus 4 bears equal 5 bears.
10. 12 bears divided by 2 equals 6 bear.
11. 1 bear and 5 bears equals 6 bears.
12. 12 bears multiplied by 2 equal 24 bears.
13. 4 quarts equals 1 gallon.
14. 1 plate multiplied by 12 equal 12 plates.
15. 1 plate divided by 4 equals $\frac{1}{4}$ plate.
16. 2 and 6 equals 8.
17. 1 child plus 6 children are 7 children.
18. 7 mice−6 mice are one mouse.

The key to the above statements is found at the end of this article.

This issue over the use of "equal" or "equals" may be an insignificant point in the teaching of arithmetic. It is readily admitted that in the kindergarten-primary area communicative expressions should have wide latitude. But, since our students are prospective teachers, we don't like to dispose of the issue too lightly, We would like to feel that at least there is a preferred way of reading the = symbol and that teachers should *know* and *use* the preferred way.

[1] The most helpful information was provided by a faculty colleague, Associate Professor Mabel L. Moberg, who referred us to NUMBERS IN ACTION, TEACHERS' EDITION, by Maurice L. Hartung, Henry Van Engen, and Catherine Mahoney. Scott Foresman and Company, New York. 1951.

Key:

1.—i	7.—i	13.—i
2.—c	8.—c	14.—i
3.—c	9.—i	15.—c
4.—i	10.—c	16.—i
5.—c	11.—i	17.—i
6.—c	12.—c	18.—c

Rules used:

1. When numbers referring to abstractions are connected with a "plus," "minus," etc., the verb should be singular as in statements 1, 2, 3, 4, 5, 6, and 7 above. The words "plus" and "minus" are used as prepositions.
2. When numbers refer to concrete objects the verb is singular or plural in accordance with the subject: singular in statements 9, 14, 15, and 17, above; plural in 8, 10, 12, 13, and 18, above.
3. When numbers are connected with an "and" the verb should always be plural as in statements 11, and 16, above. The word "and" is used as a conjunction.

EDITOR'S NOTE. Do you agree with Dr. Boyer's general principles and the application thereof to the specific exercises? Are there other areas of language useage in arithmetic in which we should seek general agreement? Many teachers have worried over "4+5 is (or) are 9." Similarly, one hears "mathematics require good thinking."

Correction

In the April, 1955 issue of *The Arithmetic Teacher* the pamphlet, *Solving Arithmetic Problems Mentally*, was incorrectly priced at 10¢. The correct price is 25¢. The pamphlet is available from Extension Service, Iowa State Teachers College, Cedar Falls, Iowa.

The Washington Meeting, Dec. 27-30, 1955

"Santa Claus is coming to town" but so are a lot of mathematics teachers for the December meeting of the National Council of Teachers of Mathematics. And they'll be bringing their families too, because at the Sheraton Park Hotel where the convention will be held, room rent is free for children under 14 who come along.

They'll be coming early too because there will be sightseeing trips to Mt. Vernon, Arlington, Smithsonian Institution, The Library of Congress etc. There will be trips to the Bureau of Standards where they will see the standard units of measure for this country, and also the electronic computer in action. A special mathematics tour of The White House has been arranged. There's an Embassy Tour too, and you can have trips to several foreign countries because the Embassy soil is considered the soil of the country owning it! The Pageant of Lights, a one-week Christmas Festival last year in Washington, was considered so outstanding that it will be repeated this year and you will want to see it. (Do bring color film.)

Of course, there will be a splendid program. And you just can't afford to miss the convention and seeing old friends and making new ones. Programs will be mailed to you in November, so do get your registrations in early before the quotas for some of the tours are filled.

The mathematics teachers of the Greater Washington area, your hosts and hostesses, will be at the gate looking down the road for you!

VERYL SCHULT
Local Chairman

Receipts and Expenditures of the

NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS for the Fiscal Year, June 1, 1954–May 31, 1955

Receipts		
Memberships and *Mathematics Teacher* subscriptions	$34,986.15	
ARITHMETIC TEACHER subscriptions	6,577.33	
Mathematics Student Journal subscriptions	5,669.26	
Sale of advertising space in *Mathematics Teacher*	3,434.25	
Sale of advertising space in ARITHMETIC TEACHER	1,169.19	
Miscellaneous	376.65	
Net return from conventions	34.01	
Sale of publications		
Yearbooks	13,980.97	
Miscellaneous	8,476.02	
TOTAL RECEIPTS		$74,703.83
Expenditures		
Washington office	$24,227.11	
President's office	1,178.06	
Mathematics Teacher	22,424.29	
ARITHMETIC TEACHER	4,474.06	
Mathematics Student Journal	3,233.71	
Contributions to NEA Building Fund	1,000.00	
Miscellaneous	2,130.19	
Printing and preparation of publications	6,870.86	
TOTAL EXPENDITURES		65,538.28
Increase in Cash Resources		$ 9,165.55

Although the above report is largely self-explanatory, comments on some of the items may aid your understanding of it. The receipts from the sale of memberships and subscriptions are given according to the old membership provisions. In the future, with members having a choice of journals (the *Mathematics Teacher* or THE ARITHMETIC TEACHER), a new break-down will be required.

It will be noted that our second largest source of income is the sale of publications. As would be expected, the best seller among the yearbooks during the past year was the latest yearbook, the 22nd. Sixty per cent of the income from the sale of yearbooks came from this one item. The best sellers among the miscellaneous publications were the five new booklets which were prepared by the Committee on Supplementary Publications and which came off the press during the months of January through April. Forty percent of the income from the sale of the miscellaneous publications came from these five booklets.

A few words should probably be said about the expenditures. The expenditures charged to the Washington Office include all costs for salaries (for a staff of five persons); office forms and supplies; mailing (except for the journals) of publications, renewal notices, and letters; membership promotion; and travel by the executive secretary. Expenditures charged to the president's office include, in addition to travel and office expenses, payments from the Special Projects Fund. The figure for each journal includes the total expense for printing, mailing, and editorial costs. The contribution to the NEA Building Fund was voted by the Board of Directors as an expression of appreciation for the many services that the NEA gives without charge to the Council. The miscellaneous expenses include travel to the Annual Meeting by the Board of Directors, committee work, and certain special authorized expenses.

The business for the year, as you will note, produced a significant increase in cash resources. This increase, however, is not necessarily profit. At the end of the fiscal year we had certain outstanding obligations for which funds needed to be reserved. We had certain funds due us, largely on unpaid invoices. We also had about $16,000 invested in yearbooks and other publications. All of our resources and liabilities must be considered in evaluating our financial condition. The executive secretary prepares semi-annually an overall Statement of Assets and Liabilities which gives us a figure called our Net Worth. During the past fiscal year our Net Worth increased approximately $7,000, thus giving us a Net Profit of that amount for our financial operations during the year. It has truly been a good year for the Council.

Readers who have further questions are invited to write to the executive secretary.

M. H. AHRENDT, *Executive Secretary*

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Twentieth, *The Metric System of Weights and Measures,* $2.00

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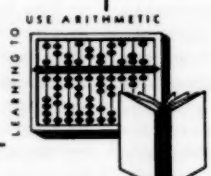

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